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Current limitation in mercury vapour discharges I. Theory

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Abstract. A theoretical description of the current limit phenomenon based on the limited ionizing ability of the electrons has been obtained. The current limit is found to be a double-valued function of pressure. This limited ionizing ability implies that positive column existence is only possible when the product of the gas density and column radius is greater than a critical value. High currents reduce the gas density to the critical value causing current limitation. Assumptions of the form of the electron velocity distribution at current limit are avoided by working with the mean free path of the neutral for ionization, λ, as a parameter. Both the ion wall current and the neutral depletion are calculated in terms of λ and, by relating ion wall current and arc current, gas density is related to arc current. Low currents reduce the plasma radius until the critical value is reached causing current limitation.

1. Introduction

It has long been known, Langmuir and Mott-Smith (1924), that an upper limit exists to the current which a low pressure arc discharge can pass. For the dc arc, current increase is achieved with slight rise in arc voltage until at a sharply defined current level the limitation phenomenon occurs. This limitation may be characterized by complete arc extinction or the arc voltage abruptly rises until the external voltage supply is exhausted. The current limit level is a function of gas pressure and tube radius but is independent of electrode effects, that is it is a positive column phenomenon.

Substantially different physical effects occur depending on whether the arc current is steady or pulsed (and if pulsed, on the time scale) and on whether the discharge tube is of simple or constricted geometry (a physical constriction in the positive column precipitates the formation of an electrostatic double sheath which strongly accelerates electrons causing localized enhancement in ionization, see Andrews and Allen 1970). In the present work the unconstricted dc discharge is considered. Attention is further confined to the ion free fall régime (Tonks and Langmuir 1929), which for mercury vapour occurs at pressures below 1 mtorr (for radius about 1 cm).

The first quantitative theoretical work in this area was that of Allen and Thonemann (1954) and extended in the work of Allen *et al.* (1963) and Caruso and Cavaliere (1964).

The ion wall current is given by

$$I_i = e\alpha n_s v_B \tag{1}$$

where e is the electronic charge, n_s is the charged particle density at the plasma boundary and α is a constant of order 1 to account for the distribution in ion energies (see Allen and Thonemann 1954) and

$$v_B = \left(\frac{kT_e}{M}\right)^{1/2}$$

is the Bohm speed where T_e is the electron temperature and M the ion mass.

The random electron current density at the sheath edge is

$$I_{er} = en_s\left(\frac{kT_e}{2\pi m}\right)^{1/2} \tag{2}$$

where m is the electron mass. Then the tube current is

$$I = \left(\frac{\beta\gamma}{\alpha}\right)\left(\frac{M}{2\pi m}\right)^{1/2} I_i \pi r_0^2 \tag{3}$$

where γ is the ratio of electron density averaged across the tube to the density at the sheath edge (J. G. Andrews 1968 private communication), β is the ratio of drift to random electron current and r_0 is the tube radius.

Now I_i/e can never exceed the flux ν_0 of neutral particles returning from the wall, hence the current limit, as given by Allen and Thonemann, is

$$I_{AT} = \left(\frac{\gamma\beta}{\alpha}\right)\left(\frac{M}{2\pi m}\right)^{1/2} \nu_0 e \pi r_0^2 \tag{4}$$

assuming that $\beta\gamma/\alpha$ is constant (the constancy of $\beta\gamma/\alpha$ as current limit is approached has been verified experimentally with the use of wall probes, Stangeby and Allen 1971).

If there are liquid Hg deposits on the tube walls then ν_0 is given by the wall temperature. If the Hg deposits are elsewhere in the system then it may be argued (Allen *et al.* 1963) that ν_0 is established by the temperature of these deposits independent of arc current (this has been verified experimentally with pressure gauges, Stangeby and Allen 1971).

According to this picture, at current limit the neutral particles are ionized even before reaching the centre of the tube. This may be objected to (Stangeby 1968) on the grounds of the limited ionizing ability of the electrons. That is, even if the mean electron energy corresponds to the maximum of the ionization cross sections, the mfp (mean free path) of the neutral for ionization (λ) may still be greater than r_0. The electrons reach the limit of their ionizing power before the condition $I_i/e = \nu_0$ is attained.

A preliminary report of the following work has already appeared, see Stangeby and Allen (1970).

2. Theory including finite λ

The effect of the finite ionizing ability of the electrons on the arc characteristics may be seen by the following qualitative argument. The ion loss rate per unit length is

$$2\pi r n_s \alpha v_B \tag{5}$$

while ion production is given by

$$\pi r^2 \bar{n}_e n g(T_e) \tag{6}$$

where n is the gas density, $\bar{n}_e = \gamma n_s$ and $g(T_e)$ is a function of the electron temperature and the ionization cross section of the discharge gas. From equations (5) and (6) one obtains the basis for the relation between T_e and nr:

$$\frac{2\alpha}{\gamma}\left(\frac{k}{M}\right)^{1/2} \frac{T_e^{1/2}}{g(T_e)} = nr. \tag{7}$$

Owing to the finite ionization cross section, $g(T_e)$ has a maximum value and hence a minimum exists to possible values of the parameter nr for existence of the positive column.

Poletaev (1951) has provided the experimental value for Hg vapour

$$(nr)_{\min} = C_1 = 3{\cdot}9 \times 10^{16}\,\mathrm{m}^{-2}. \tag{8}$$

The value of C_1 is discussed in Appendix 2.

As equation (7) is explicitly independent of I this latter theory does not directly provide a current limit theory. However, sufficiently high currents cause neutral gas depletion hence $n = n(I)$, effectively. Also at very low currents the wall sheath grows, reducing the plasma radius r, hence $r = r(I)$, effectively.

2.1. *High-current régime*

Current limitation is assumed to occur when I reaches I_L making

$$\bar{n}(I_L)r_0 = C_1$$

where $\bar{n}(I)$ is the neutral density averaged across the tube cross section and $\bar{n}(I) < n_0$, the density at $I = 0$. Since the electron distribution at current limit $f_L(\boldsymbol{v}_e)$, is strongly non-Maxwellian, calculations of $\bar{n}(I)$ which depend on assumptions of the form of the distribution will be subject to uncertainty. This problem may be avoided by working with λ—the mfp for the neutral for ionization—and calculating

(i) $I_i(\lambda)$ and then using equation (3), $I(\lambda)$

(ii) $n(r, \lambda)$ the neutral density as a function of λ and radius and thus $\bar{n}(\lambda)$ and from $I(\lambda)$ and $\bar{n}(\lambda)$ one obtains $\bar{n}(I)$. In this way the calculation of $\lambda\{n_e, f_L(\boldsymbol{v}_e)\}$ is avoided. Of course $\lambda \propto v_N$ where v_N is the neutral speed and a calculation based on the distribution of neutral speeds is straightforward in principle but for simplicity we shall assume that all the neutrals move with their average speed $\bar{c}$. Also λ is assumed to be independent of r thus λ is averaged across the tube diameter in effect.

To calculate $I(\lambda)$ consider the flux of neutral particles at an element of wall surface in the absence of a discharge. The emitted (or received) flux density according to the cosine law is

$$\int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi} \bar{c}\cos\theta\, n_0 \frac{d\Omega(\theta, \phi)}{4\pi} \tag{9}$$

where θ and ϕ are the normal and azimuthal angle respectively and the elemental solid angle is

$$d\Omega(\theta, \phi) = \sin\theta\, d\theta\, d\phi.$$

By integration over θ and ϕ the total emitted flux density is

$$\nu_0 = \tfrac{1}{4}n_0\bar{c}.$$

In the presence of a discharge the probability that an atom reaches the element s is given by $\exp\{-l(\theta, \phi)/\lambda\}$, so that the received flux density is modified to

$$\int\int \bar{c}\cos\theta \exp\left(-\frac{l(\theta, \phi)}{\lambda}\right) n_0 \frac{d\Omega}{4\pi} \tag{10}$$

where $l(\theta, \phi)$ is the distance from the sample point to the wall emission point at angle (θ, ϕ)—see figure 1—(rectilinear motion is assumed) and n_0 is now *defined* by

$n_0 = 4\nu_0/\bar{c}$. From figure 1 one finds

$$l(\theta, \phi) = \frac{2r_0 \cos\theta}{\cos^2\theta + \sin^2\theta \sin^2\phi}. \tag{11}$$

From conservation of heavy particle flow

$$I_1 = \tfrac{1}{4}\bar{c}n_0 \frac{e}{\pi} \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} \cos\theta \sin\theta \left\{1 - \exp\left(-\frac{l(\theta, \phi)}{\lambda}\right)\right\} \mathrm{d}\theta\, \mathrm{d}\phi. \tag{12}$$

Thus relating I and I_1 by equation (3) one obtains $I(\lambda)$. This relation obtained by numerical integration is given in figure 2. The abscissa is $I_1/\nu_0 e$ or equivalently I/I_{AT}. Note that the Allen–Thonemann limitation occurs when $I_1 = \frac{1}{4}n_0\bar{c}e$, that is

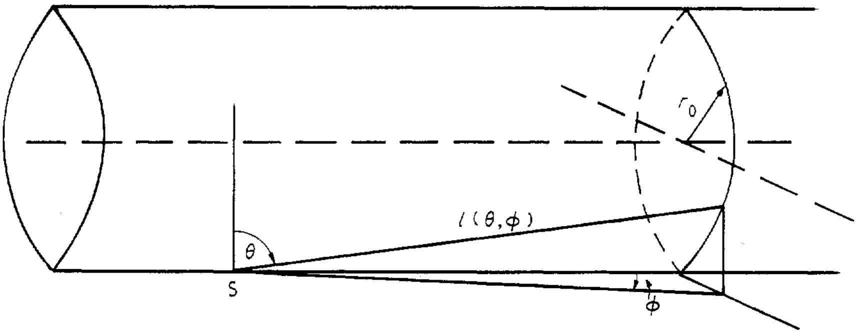

Figure 1. Neutral flux into wall point S.

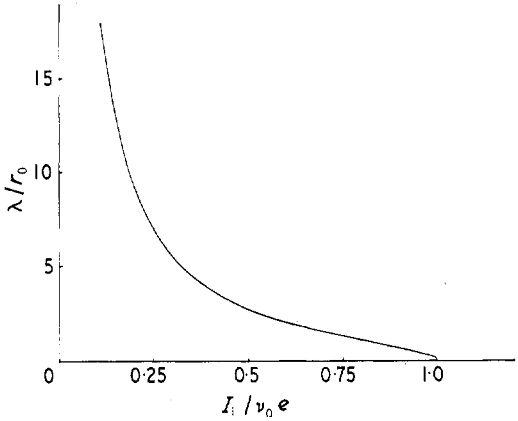

Figure 2. Neutral mfp as a function of ion wall current.

when $\lambda \to 0$ independent of starting pressure. In the present work it is postulated that for $\lambda(I_{\mathrm{L}}) > 0$ the average neutral density is reduced to n_C causing limitation where

$$n_C = C_1/r_0.$$

To obtain the neutral density distribution in the discharge volume as a function of λ, that is $n(r, \lambda)$, consider figure 3. In the absence of a discharge the elemental area dA_S at the point S on the wall emits

$$\bar{c}\cos\psi\, n_0 \frac{d\Omega(\psi, \xi)}{4\pi}\, dA_S \tag{13}$$

neutrals/s into the elemental cone $d\Omega(\psi, \xi)$ where ψ and ξ are the normal and azimuthal angles at S. Hence the flux density across the elemental surface at P, namely

$$dA_P = y^2\, d\Omega(\psi, \xi)$$

is

$$\frac{\bar{c}\cos\psi\, n_0}{4\pi y^2}\, dA_S \qquad \text{particles/m}^2\,\text{s}$$

where $y = |SP|$. This contribution to the density at P is therefore

$$dn(r, \lambda) = \frac{n_0 \cos\psi}{4\pi y^2}\, dA_s.$$

In the presence of a discharge this is modified to

$$dn(r, \lambda) = \frac{n_0}{4\pi y^2} \exp\left(-\frac{y}{\lambda}\right) \cos\psi\, dA_s. \tag{14}$$

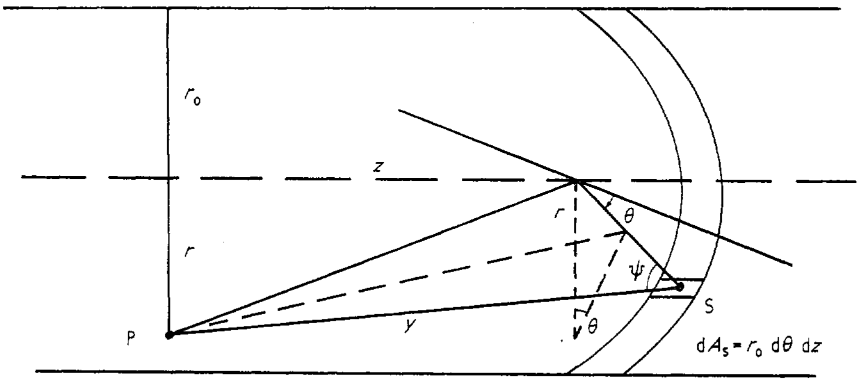

Figure 3. Neutral flux received at volume point P from S. The construction illustrates the relation $r_0 = y\cos\psi + r\sin\theta$.

From figure 3 one may obtain the relations

$$r_0 = y\cos\psi + r\sin\theta$$
$$y^2 = r_0{}^2 + r^2 + z^2 - 2r_0 r\sin\theta$$
$$dA_s = r_0\, d\theta\, dz.$$

By using these relations and normalizing with $\rho = r/r_0$ and $x = z/r_0$ one obtains

$$n(\rho, \lambda) = \frac{n_0}{\pi} \int_{x=0}^{\infty} \int_{\theta=-\pi/2}^{\pi/2} \frac{(1-\rho\sin\theta)\exp\{-(r_0/\lambda)(1+\rho^2+x^2-2\rho\sin\theta)^{1/2}\}}{(1+\rho^2+x^2-2\rho\sin\theta)^{3/2}}\, dx\, d\theta. \tag{15}$$

For $\lambda \to \infty$, $n(\rho) \to n_0$ for all ρ, as required.

This integral has been obtained by numerical integration and is shown in figure 4. (Convergence for $\rho = 1$ is slow using equation (15) and an alternative formula for $n(\rho, \lambda)$ may be used here—see Appendix 1).

From the values of $n(\rho, \lambda)$ one obtains the average neutral density across the tube

$$\bar{n}(\lambda) = \frac{1}{\pi}\int_0^1 n(\lambda, \rho)2\pi\rho \, d\rho. \qquad (16)$$

The function $\bar{n}(\lambda)$ is given in table 1.

The critical $\lambda(I_L)$ is such that $\bar{n}\{\lambda(I_L)\} = n_C$. One thus obtains $I_L(p_0)$ the current limit as a function of starting pressure as shown in figure 5. In Appendix 3 it is shown

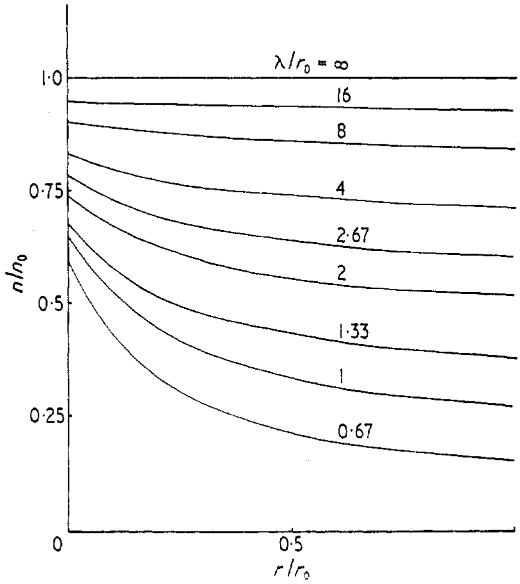

Figure 4. Radial variation of neutral density with neutral mfp as parameter.

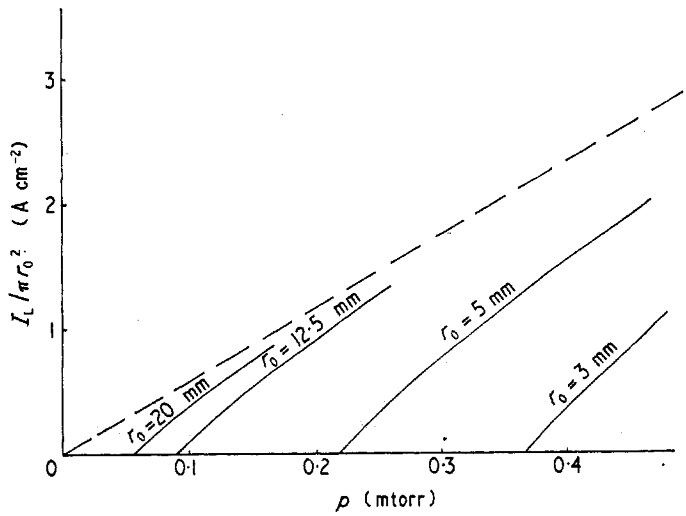

Figure 5. Current limit (high-current régime) as a function of starting pressure. $(\gamma\beta/\alpha) = 1$. Broken line gives $I_{AT}/\pi r_0^2$.

Table 1. Average neutral density as a function of the mfp

λ/r_0	$\bar{n}(\lambda/r_0)/n_0$
∞	1
16	0·940
8	0·874
4	0·763
2·67	0·679
2	0·608
1·33	0·496
1	0·418
0·67	0·311

that I_L/r_0 and n_0r_0 are similarity variables and accordingly the information in figure 5 may be compactly presented as in figure 6.†

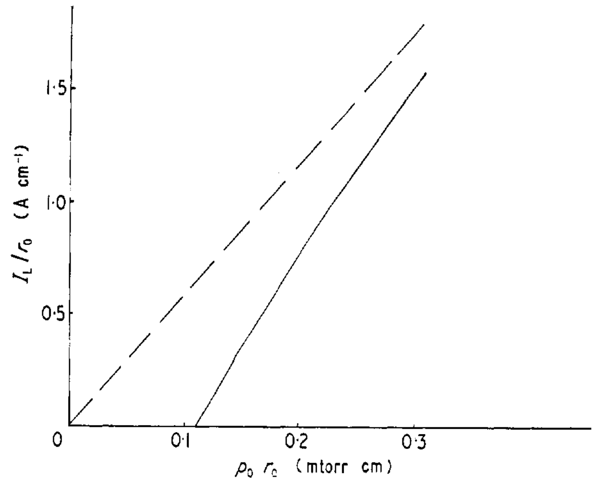

Figure 6. High-current régime limit showing similarity relation between I_L/r_0 and p_0r_0. $(\gamma\beta/\alpha) = 1$. Broken line gives I_{AT}/r_0.

A plot of $\lambda(I_L)$—see figure 7—reveals that

$$\left(\frac{I_L}{\pi r_0^2}\right)\lambda(I_L) = \text{constant}$$

which is to be expected as

$$\lambda(I_L) = \frac{\bar{c}}{\bar{n}_e \langle v_e Q_i \rangle_{max}} \tag{17}$$

where $\langle v_e Q_i \rangle_{max}$ is the neutral ionization coefficient corresponding to maximum electron ionization efficiency and

$$I_L = \pi r_0^2 \bar{n}_e e\beta \left(\frac{kT_{e_L}}{2\pi m}\right)^{1/2} \tag{18}$$

† Appreciation is expressed to Dr. R. N. Franklin for suggesting the similarity relation used in figure 6.

thus

$$\left(\frac{I_L}{\pi r_0^2}\right)\lambda(I_L) = \frac{e\beta\left(\frac{kT_{e_L}}{2\pi m}\right)^{1/2}\bar{c}}{\langle v_e Q_i\rangle_{max}}. \tag{19}$$

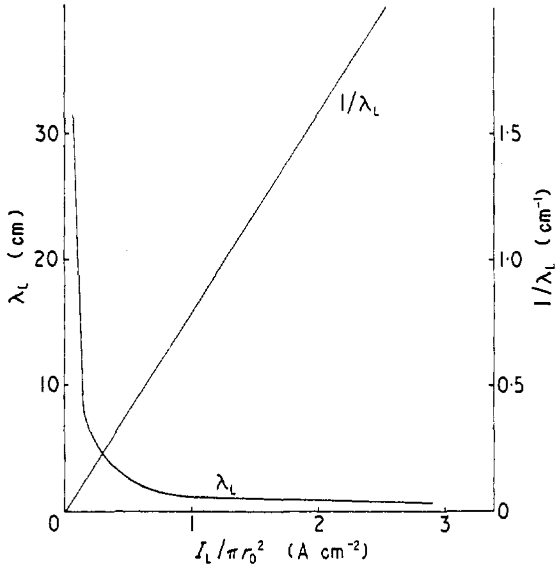

Figure 7. λ_L and $1/\lambda_L$ against high current régime limit. $(\gamma\beta/\alpha) = 1$.

2.2. *Lower current branch*

For sufficiently small currents n will be essentially unaffected by the current and will be equal to n_0, however the radius of the plasma (i.e. of the conducting region) will decrease owing to growth in the wall sheath $r = r(I)$ in effect. Thus current limit is assumed to occur when I reaches I_L making

$$r(I_L)n_0 = C_1. \tag{20}$$

The radius is given by

$$r(I_L) = r_0 - C_2\lambda_D \tag{21}$$

where λ_D is the Debye length at current limit and C_2 gives the sheath thickness in Debye lengths. For a Maxwellian electron distribution

$$\lambda_D = \left(\frac{\epsilon_0 kT_e}{n_e e^2}\right)^{1/2}. \tag{22}$$

In the present case $f_L(\boldsymbol{v}_e)$ is non-Maxwellian, however it is assumed that $f_L(\boldsymbol{v}_e)$ is independent of r_0, n_e, etc., hence

$$\lambda_D^2 n_e = C_3 \tag{23}$$

where C_3 is a constant

Combining equations (1) and (3) gives

$$I = C_4 n_e r^2 \tag{24}$$

with

$$C_4 = \beta\left(\frac{kT_e}{2\pi m}\right)^{1/2}\pi e$$

where T_e will be considered as an effective electron temperature.

Thus combining equations (20), (21), (23) and (24) gives

$$I_L(n_0) = \frac{C_1{}^2 C_2{}^2 C_3 C_4}{(r_0 n_0 - C_1)^2}. \tag{25}$$

Approximate values for the constants for mercury as the operating gas are given in Appendix 2. For $r_0 = 5$ mm and 12·5 mm the upper and lower current branches

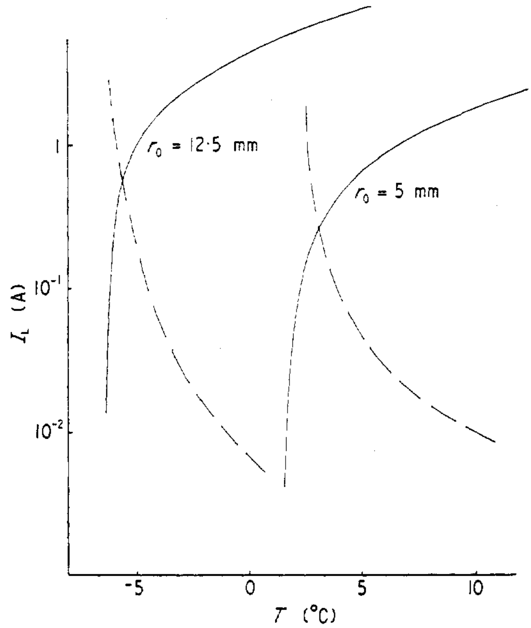

Figure 8. High and low current limit as a function of Hg condensing temperature. Full line, high régime; broken line, low régime. $(\gamma\beta/\alpha) = 1$.

are shown in figure 8. No attempt has been made to establish theoretical values for the transition between the two branches owing to the uncertainty in the constants.

3. Conclusions

A theoretical description of the current limit phenomenon based on the limited ionizing ability of the electrons has been obtained. The current limit is found to be a double-valued function of pressure. This limited ionizing ability implies that positive column existence is only possible when the product of the gas density and column radius is greater than a critical value. High currents reduce the gas density and low currents reduce the plasma radius until the critical value is reached causing current limitation.

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One of us (P.C.S.) is indebted to Shell Canada Ltd and NRC (Canada) for personal support.

Appendix 1. Alternative formula for $n(\rho, \lambda)$

In the absence of a discharge for any point P in the volume the portion of the neutral density at P which arrived via the elemental solid angle $d\Omega(\theta, \phi)$ is

$$n_0 \frac{d\Omega(\theta, \phi)}{4\pi}$$

(P is the origin for θ, ϕ). In the presence of a discharge this becomes

$$n_0 \frac{d\Omega(\theta, \phi)}{4\pi} \exp\left(-\frac{l_{PW}(\theta, \phi)}{\lambda}\right)$$

where l_{PW} (θ, ϕ) is the distance from P to the wall at angle (θ, ϕ). Hence

$$n_P = n(\rho, \lambda) = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} n_0 \frac{d\Omega(\theta, \phi)}{4\pi} \exp\left(-\frac{l_{PW}(\theta, \phi)}{\lambda}\right). \qquad (A1)$$

If P is just at the wall then half the particles arrive at P directly from the wall without attenuation and the rest as per equation (A.1). Hence

$$n(1, \lambda) = \tfrac{1}{2} n_0 + \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} n_0 \frac{d\Omega}{4\pi} \exp\left(-\frac{l(\theta, \phi)}{\lambda}\right)$$

where

$$l(\theta, \phi) = \frac{2r_0 \cos\theta}{\cos^2\theta + \sin^2\theta \sin^2\phi}$$

as in equation (11). Values of $n(1, \lambda)$ obtained by numerical integration can be found from figure 4.

Appendix 2. Values of the constants for mercury vapour

(*a*) C_1: Poletaev (1951), assuming a Maxwellian distribution for the electrons has calculated that

$$(rn)_C = 2{\cdot}8 \times 10^{16}\ \mathrm{m}^{-2}$$

$$(pr)_C = 0{\cdot}079\ \mathrm{mtorr\ cm}$$

(corresponding to a T_{e_L} of about 25 eV) compared with an experimental value of

$$(rn)_C = 3{\cdot}9 \times 10^{16}\ \mathrm{m}^{-2}$$

$$((pr)_C = 0{\cdot}11\ \mathrm{mtorr\ cm}).$$

The disagreement between the two figures is not excessive considering that:

(i) the electron distribution at current limit is substantially non-Maxwellian owing to strong electron drift and depletion of electrons due to excitation and ionization.

(ii) Poletaev uses a mathematical approximation to the ionization cross section data of Bleakney (1930). This approximate cross section is not in complete agreement with the more precise work of Nottingham (1939)

(iii) no account is taken of the loss rate of Hg^{2+} ions (which reach the sheath edge with speeds $\sqrt{2}$ times greater than the Hg^+ ions) despite the fact that the ionization cross section for Hg^{2+} starts at 29 eV.

Accordingly the experimental value is taken so that

$$C_1 = 3{\cdot}9\times10^{16}\,\mathrm{m}^{-2}.$$

(*b*) C_2: For very small electron desnsities the division of the positive column into plasma and sheath regions becomes increasingly artificial. However, an approximate value for the sheath thickness in Debye lengths is given by Self (1963) for the low-current régime considered here as

$$C_2 \simeq 9.$$

This is not in conflict with the value $C_2 \simeq 10$ which may be estimated from the work of Tonks and Langmuir (1929).

(*c*) C_3: The calculation of the Debye length itself is rendered difficult by the non-Maxwellian nature of the electron distribution at current limit; however, recent experimental results, Stangeby and Allen (1971), indicate that the effective T_e at current limitation is approximately $T_{e_L} \simeq 10$ eV and accordingly

$$\lambda_D{}^2\, n_e = \frac{\epsilon_0 k T_{e_L}}{e^2} = C_3 \simeq 5{\cdot}5\times10^8\,\mathrm{m}^{-1}.$$

(*d*) C_4: The calculation of C_4 is also dependent on T_{e_L} but only as $T_{e_L}{}^{1/2}$, hence from equation (24), and setting $\beta = 1$,

$$C_4 = \beta\left(\frac{kT_{e_L}}{2\pi m}\right)^{1/2}\pi e \simeq 2{\cdot}7\times10^{-13}\,\mathrm{A\,m}^{-1}.$$

Appendix 3. I_L/r_0 and n_0r_0 as similarity variables (high-current régime)

Four equations are used to calculate $I_L(n_0, r_0)$:

$$I_{i_L} = \tfrac{1}{4}\bar{c}n_0\frac{e}{\pi}\int\int \mathrm{d}\theta\,\mathrm{d}\phi\sin\theta\cos\theta\left[1-\exp\left\{-\left(\frac{r_0}{\lambda_L}\right)\left(\frac{2\cos\theta}{\cos^2\theta+\sin^2\theta\sin^2\phi}\right)\right\}\right] \quad \text{(A2)}$$

$$I_L = \left(\frac{\gamma\beta}{\alpha}\right)\left(\frac{M}{2\pi m}\right)^{1/2}\pi r_0{}^2 I_{i_L} \quad \text{(A3)}$$

$$\bar{n}_L = \frac{n_0}{\pi^2}\int\int\int\frac{\mathrm{d}\rho\,\mathrm{d}x\,\mathrm{d}\theta\,2\pi\rho(1-\rho\sin\theta)\exp\{-(r_0/\lambda_L)(1+\rho^2+x^2-2\rho\sin\theta)^{1/2}\}}{(1+\rho^2+x^2-2\rho\sin\theta)^{3/2}} \quad \text{(A4)}$$

$$\bar{n}_L = \frac{C_1}{r_0}. \quad \text{(A5)}$$

Thus, from equation (A2),

$$I_{i_L} = \tfrac{1}{4}\bar{c}\,n_0 f\left(\frac{r_0}{\lambda_L}\right) \quad \text{(A6)}$$

where f is a function of r_0/λ_L only; from equations (A3) and (A6)

$$\frac{I_L}{r_0} = r_0 n_0\left(\frac{\gamma\beta}{\alpha}\right)\left(\frac{M}{2\pi m}\right)^{1/2}\frac{\pi}{4}\bar{c}f\left(\frac{r_0}{\lambda_L}\right)$$

so that

$$\left(\frac{I_L}{r_0}\right) = (r_0 n_0) g\left(\frac{r_0}{\lambda_L}\right) \tag{A7}$$

where g is a function of r_0/λ_L only; from equation (A4)

$$\bar{n}_L = n_0 h\left(\frac{r_0}{\lambda_L}\right) \tag{A8}$$

with h a function of r_0/λ_L only.

Combining (A5) and (A8)

$$h\left(\frac{r_0}{\lambda_L}\right) = \frac{C_1}{(n_0 r_0)}$$

hence

$$\frac{r_0}{\lambda_L} = h^{-1}\left(\frac{C_1}{r_0 n_0}\right) \tag{A9}$$

and from (A7)

$$\frac{r_0}{\lambda_L} = g^{-1}\left(\frac{I_L/r_0}{r_0 n_0}\right) \tag{A10}$$

so that from (A9) and (A10)

$$\left(\frac{I_L}{r_0}\right) = (r_0 n_0) g\left\{h^{-1}\left(\frac{C_1}{r_0 n_0}\right)\right\}. .$$

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